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DISTRIBUTION OF MASSES IN GALAXIES ACCORDING TO DATA ON RADIAL VELOCITIES

by

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SUMMARY

Two new ways are proposed for the solution of the Burbidge and Prendergast equation. The results of calculation of the mass for 12 galaxies and of the density for 4 galaxies are given. The tables and the diagrams presented show considerable departures from the results of other authors.

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The data on radial velocities of points situated at various distances from the center of a galaxy allow us to determine the law of mass distribution in them.

Wise and Mayall worked out in 1942 [1] a method of mass and density determination in the assumption of total flattening of galaxy body; in it the expansion of density in series by powers relative to distance from the center was applied. The authors used this method to determine, in particular, the mass of NGC 224 $(9.5 \cdot 10^{10}\ M_{\odot}$ at the distance of 210 kps).

An important step forward was performed in 1959 by Burbidge and Prendergast [2], in whose method galaxies are not considered entirely flat, but in which it is assumed that the surfaces of constant density are ellipsoids of revolution with constant eccentricity. At the same time, expansions are also utilized of density, circular velocity and its square, in series by powers of the distance from the center. According to our opinion, the method proposed in [2] may be improved in several directions because of the following.

1. A finite number of terms are utilized in the expansions (in case of circular velocity we refer to parameters). The selection of this number is determined by the extent the theoretical curve for the velocity passes through the points of observations, which has to be estimated subjectively. As to the

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variation of the number of parameters, it generally leads to a significant range of masses and densities. Moreover, at a generally satisfactory passage through the points of observations the theoretical curves of velocity give a considerable smoothing of local velocity functions (and also of density) as this was noted by P. Pishmish [17].

2. It is desirable to deny ourselves the assumption of eccentricity constancy of ellipsoids, that is, of constant density surfaces.

3. At sufficient galaxy inclinations to the visual ray it is difficult to estimate the magnitude of contraction. That is why in the given case it is more appropriate to evaluate it by the dispersion of velocities.

4. When determining the densities and masses, the systematic accounting of dispersion is unavailable, except for the estimates of [2].

5. The method requires significant computation work.

Brandt [3] found in 1960 a simple solution of the Burbidge integral equation for the case of zero sphericity.

The Burbidge equation was utilized in the work by Agekyan and Yakovleva [4] for the determination of masses of galaxies directly by way of numerical integration over observation points, without drawing the mean curve of velocities. When searching for the correction for sphericity, the assumption is made that the density is inversely proportional to the square of the distance from the center.

The Agekyan work (page 641 of ref.[5]) complements the indicated methods by the determination of the density of a uniform dispersing nucleus by its angular velocity and contraction.

But, on the whole, a single approach is required for the consideration of a galaxy, that is, a systematic accounting of velocity dispersion and a search of general solutions (for the nucleus in the extranuclear region), moreover as simple and objective as possible.

It is proposed in this work to resolve for the moment only a part of this problem: we propose a certain improvement of the methods of [2] and [4].

Let us consider the Burbidge & Prendergast integral equation [2]:

$$v^2(R) = 4\pi G c \int_0^R \frac{\rho(a)\, a^2\, da}{\sqrt{R^2 - e^2 a^2}}, \tag{1}$$

where v(R) is the circular rotation velocity at the distance R from the center of the galaxy in its plane; $\rho(a)$ is the density as a function of the major semiaxis a of the ellipsoid of revolution, which is the surface of constant-density; e is the eccentricity of ellipsoid's meridional cross-section;

$e = \sqrt{1 - e^2}$ is the sphericity; G is the gravitational constant; e and c are constant.

Let us multiply both parts of (1) by

$$\frac{RdR}{\sqrt{A^2 - R^2}},$$

integrate between the limits from 0 to A, and change the order of integration. We shall obtain

$$4\pi c \int_0^A \rho(a) a^2 da \int_a^A \frac{RdR}{\sqrt{(A^2 - R^2)(R^2 - e^2 a^2)}} = \frac{1}{G}\int_0^A \frac{v^2(R) RdR}{\sqrt{A^2 - R^2}}. \qquad (2)$$

The inner integral in the left-hand part of (2) is computed, wherefrom

$$4\pi c \int_0^A \rho(a) a^2 da = \frac{2}{\pi G}\int_0^A \frac{v^2(R) RdR}{\sqrt{A^2 - R^2}} + 8c \int_0^A \rho(a) a^2 \operatorname{arc\,tg} \frac{ca}{\sqrt{A^2 - a^2}} da.$$

Inasmuch as the mass, comprised within the limits of the ellipsoid of revolution with major semiaxis A, is

$$M(A) = 4\pi c \int_0^A \rho(a) a^2 da, \qquad (3)$$

we have

$$M(A) = \frac{2}{\pi G}\int_0^A \frac{v^2(R) RdR}{\sqrt{A^2 - R^2}} + 8c \int_0^A \rho(a) a^2 \operatorname{arc\,tg} \frac{ca}{\sqrt{A^2 - a^2}} da. \qquad (4)$$

For small c the integral

$$I = \int_0^A \rho(a) a^2 \operatorname{arctg} \frac{ca}{\sqrt{A^2 - a^2}} da \qquad (5)$$

is a small quantity by comparison with $\frac{M(A)}{8c}$ Indeed, at ρ = const, we obtain

$$I = \frac{\pi}{6} A^3 \rho \left[1 - \frac{2}{\pi}\left(\frac{\arcsin e - ce}{e^3}\right)\right] = \frac{M(A)}{8c}\left(\frac{4}{\pi} c - \frac{3}{2} c^2 + \frac{16}{3\pi} c^3 - \dots\right) \qquad (6)$$

The condition of smallness of I will be so much the more satisfied for a density decreasing with the distance, for in that case

$$I = \frac{\pi}{6} A^3 \bar{\rho}_1 \left(\frac{4}{\pi} c - \frac{3}{2} c^2 + \dots\right), \quad \frac{M(A)}{8c} = \frac{\pi}{6} A^3 \bar{\rho}_2, \qquad (6)$$

whereupon

$$\bar{\rho}_1 < \bar{\rho}_2$$

$\bar{\rho}_1$ and $\bar{\rho}_2$ being the mean values of densities taken out of integral signs in respectively the expressions (5) and (3), as a consequence of which

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$$I = \frac{M(A)}{8c} \cdot \frac{\bar{\rho}_1}{\rho_2} \left(\frac{4}{\pi} c - \frac{3}{2} c^2 + \ldots \right) < \frac{M(A)}{8c} \left(\frac{4}{\pi} c - \frac{3}{2} c^2 + \ldots \right).$$

At the same time the local positive density gradients do not change substantially the picture, for their influence is equi alent to the introduction of restricted numerical multipliers, differing little from unity, into expressions M(A)/8c and I. Therefore, the zero approximation for the mass will be

$$M_0(A) = M(A,\ c = 0) = \frac{2}{\pi G} \int_0^A \frac{v^2(R)\,R\,dR}{\sqrt{A^2 - R^2}} \tag{7}$$

that is, the Brandt solution [3] for a plane case, or upon integration by parts

$$M_0(A) = \frac{4}{\pi G} \int_0^A v(R) \sqrt{A^2 - R^2}\, dv(R). \tag{8}$$

The corresponding approximation for the density is

$$\rho_0(a) = \frac{1}{4\pi c a^2} \cdot \frac{dM_0(a)}{da}. \tag{9}$$

Substituting ρ_0 in (4), and then also the subsequent approximations of ρ we shall obtain the final expression for the mass

$$M(A) = M_0(A) + \frac{2}{\pi} \int_0^A \frac{dM_0(a)}{da} \operatorname{arc\,tg} \frac{ca}{\sqrt{A^2 - a^2}}\, da +$$
$$+ \frac{2}{\pi} \int_0^A \frac{d}{da} \left(\frac{2}{\pi} \int_0^a \frac{dM_0(t)}{dt} \operatorname{arc\,tg} \frac{ct}{\sqrt{a^2 - t^2}}\, dt \right) \operatorname{arc\,tg} \frac{ca}{\sqrt{A^2 - a^2}}\, da + \ldots \tag{10}$$

For the particular case of solid-body rotation we assume in the first approximation ρ = const, and, utilizing (4) and (6) we shall find precisely

$$M(A) = \frac{2}{GK} \int_0^A v(R) \sqrt{A^2 - R^2}\, dv(R) \tag{11}$$

or

$$M(A) = \frac{2}{3} \cdot \frac{\omega^2 A^3}{GK}, \tag{12}$$

where

$$K = \frac{\arcsin e - ce}{e^3};$$

ω being the angular velocity.

As was shown by calculations with utilization of (10), for small sp ricities ($c \lesssim 1/5$) the subsequent to preceding term ratio in the expansion (10) is about equal to c. This is why not more tham three-four terms in expansion (10) are required for $c \lesssim 1/5$) when computing M with a precision to, e. g. 1% (because of other kind of errors, a greater precision is not necessary).

For greater (and, generally speaking, for any) values of sphericity it is possible to propose still one more method of seeking density and mass by using Eq.(1).

Let us substitute the integral in Eq.(1) by a finite sum, changing parameter R from the least possible to still greater values. This provides the possibility to obtain linear algebraic equations relative to $\rho(a)$. But inasmuch as the velocity enters in Eq.(1) in the second power, the computations directly by observation points will not provide the required precision. An objective way must be obtained for the search of centroid velocities. To that effect we shall make use of one of the possible variants. Let us denote by $\bar{v}(R)$ the velocity of the centroid and write the equality

$$\int_0^R \bar{v}(t)\,dt = \sum(R), \tag{13}$$

where $\Sigma(R)$ is the value of the integral in the left-hand part, integrated numerically over the observation points. Then, differentiating equality (13) with respect to R, we shall find

$$\bar{v}(R) = \frac{d\sum(R)}{dR}. \tag{14}$$

As was shown by computations, the scattering of the points in the graph for $\Sigma(R)$ is substantially less than that in the graph for $v(R)$, and the curve for $\Sigma(R)$ is drawn confidently, and this is why the curve for $\bar{v}(R)$ is found with as much assurance. For the transition from the visible mass M_V, density ρ_V and distance from the center, R_V to the real M, ρ and R, it is necessary to take into account the inclination.

It is easy to find that

$$M = M_v\left(\sqrt{1+\frac{\mathrm{tg}^2\beta}{\cos^2 i}}\right)^3\frac{\cos\beta}{\sin^2 i},$$

$$\rho = \rho_v\,\frac{1}{\sin^2 i\cos^2\beta},$$

$$R = R_v\sqrt{1+\frac{\mathrm{tg}^2\beta}{\cos^2 i}}\,\cos\beta,$$

where i is the angle between the visual ray and the perpendicular to the plane of the galaxy; β is the projection on the pictorial plane of the angle between the line of nodes and the slit, in which spectra are obtained.

In order to illustrate the method we proceeded with calculations for 12 galaxies having as great a diversification in the shapes of rotational curves as is possible.

Expressions (8) and (11) and also (10) were integrated numerically without drawing the median curve v(R), the separate irregularities in the behavior of which being then compensated.

In case of solid (consistent) rotation the integration of (11) must be performed when the scattering of the points in the graph for v(R) is great and the inclination is difficult to establish; in the opposite case the calculations may be conducted by a more convenient relation, namely, (12).

The calculations of masses by formulas (8), (10) — (12) were controlled by the second method (by numerical solution of Eq.(1)), which should be applied in the first place for the computation of densities and for masses at substantial sphericites ($c \gtrsim 1/5$). The results of computations are compiled in Table 1 (see also Figures 1–4)

TABLE 1

NGC	Scale	A	c	Prev. mass determ. (in $10^{10} M_\odot$)	$M \cdot 10^{-10} M_\odot$
224	1) 1″ = 1 nc 2) 1″ = 2.2nc	150′	0.2	1) 9.5 [1] 1) 9.7 [5, 645) 1) 10.2 [16] 1) 8.3 [4]	1) 8.7 [5 p. 640 2) 19.1
681	1″ = 111.7 nc	47″	1/4 1/5 1/8	1.64–2.09 1.81–2.05 [15] 1.91–1.98	2.20 2.14 [15] 2.07
1068	1″ = 77.5 nc	25″	0.2	2.7 [6] 0.80 [4]	1.14 [6]
1097	1″ = 77.9 nc	10″.25	1/3	0.5–1.3 [7] 1.36 [4]	0.955 [7]
2903	1″ = 29.1 nc	140″	0.1	3.7 [8] 7.14 [4]	3.78 [8]
3504	1″ = 96 nc	50″	1/2 1/8	0.25 [9] 0.84 [4] 0.91	0.96 [9] 0.825
3556	1″ = 50 nc	175″	0.2 0.15 0.1	1.3–1.7 1.2–1.6 [10] 0.85 [4] 1.1–1.5	1.825 1.73 [10] 1.625
4826	1″ = 39 nc	50″	1/4 1/15	0.989–1.32 [13] 0.886–1.11	1.12 [13] 1.018
5055	1″ = 50 nc	200″	1/15	7.6 [11] 7.98 [4]	7.755 [11]
6503	1″ = 76 nc	60″	1/4 1/10	0.770 * 0.655	0.92 [18] 0.798
7331	1″ = 70 nc	143″	1/4 1/15	7.8–9.5 [14] 7.7–8.2	9.56 [14] 8.59
7479	1″ = 170.7 nc	54″	1/6	2.22 [12] 6.71 [4]	5.08 [12]

* These data were computed on the basis of densities in ref. [18].

In Table 1 A denotes the major semiaxis of the ellipsoid, within the bounds of which the mass has been determined. The computed values of masses (for different values of sphericity c) and the references from which the observational material was borrowed are given in the last column.

The figures and Table 1 show notable discrepancies with the results of other authors, and in particular, the absence of ambiguity in the determination of masses and densities; they show also a lesser smoothness in local density

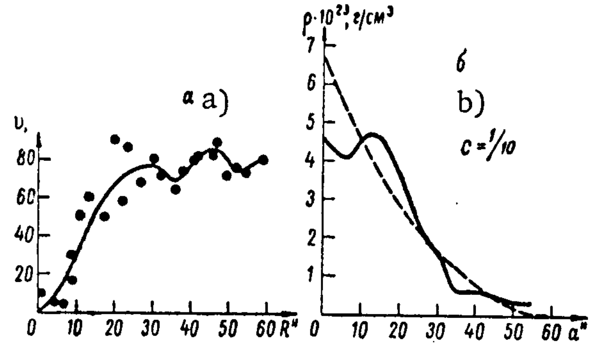

Fig.1. NGC 6503

a) radial velocities; b) computed density for c = 1/10 (solid line);dashes- Burbidge data

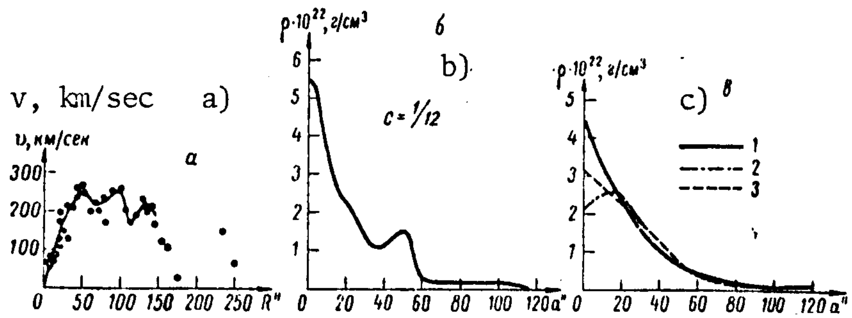

Fig.2. NGC 7331

a) radial velocities; b) computed density for c = 1/12; c) computed density for c = 1/12; Burbidge data: 1) by 4 parameters, 2) by 6 parameters, 3) by 7 parameters

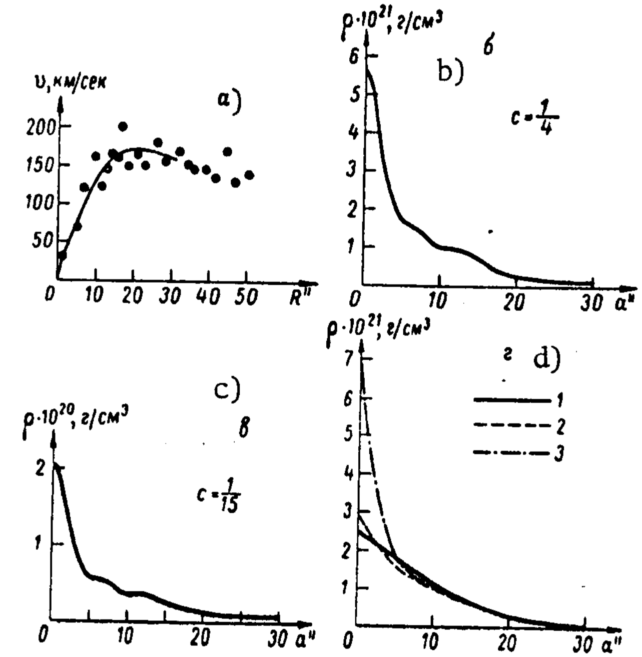

Fig.3. NGC 4826

a) radial velocities; b) and c) computed density (if we construct graphs of relative density for c = 1/4 and c = 1/15, we may be convinced that they fully coincide); d) computed density for c = 1/4; Burbidge Data: 1) by 3 parameters, 2) by 4 parameters, 3) by 7 parameters

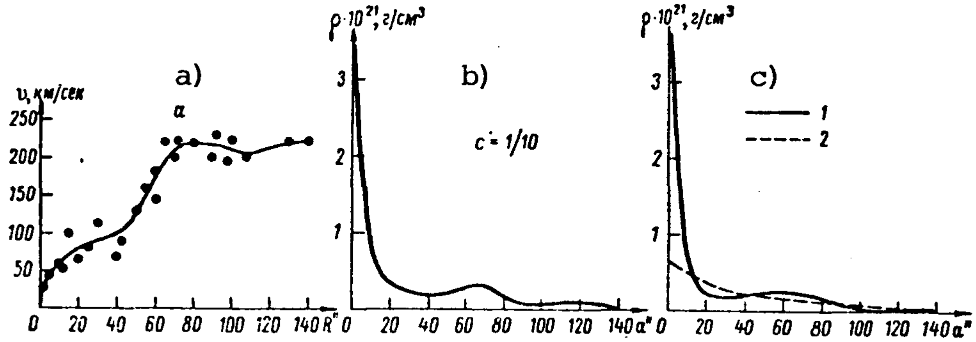

Fig.4. NGC 2903

a) radial velocities; b) computed density for c = 1/10; c) computed density for c = 1/10; Burbidge data: 1) by 7 parameters; 2) by 5 parameters

fluctuations. As typical examples, we brought out in the figures the computations of density for 4 out of 12 galaxies. The dots indicate the observation data, not corrected for inclination, and the median curve, passing through them, has been computed by formula (14).

**** THE END ****

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